

Revision Guide

Fractions

Ratio

Factors

Simultaneous Equations



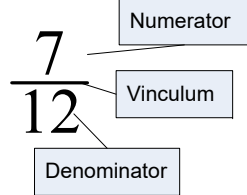
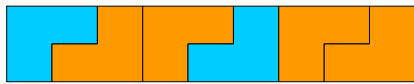
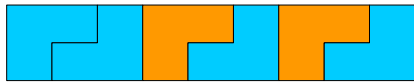
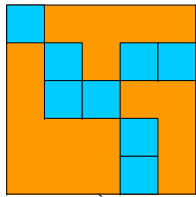
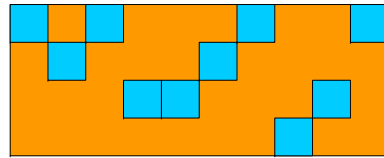
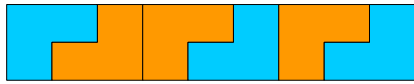
Fractions

A What fraction of the following shapes are blue?



Count the number of blue parts. That becomes the **numerator**.

Count the **total** number of parts. That becomes the **denominator**.



Sometimes, you might need to draw your own lines in.

C Find fractions of amounts Find $\frac{13}{20}$ of 65

Look for common factors to cancel down numbers and make life easier... 5 is one

$$\frac{13}{20} \times \frac{65}{1} = \frac{169}{4} = 42\frac{1}{4}$$

"of" means times

Top x top. Bottom x bottom.

$169 \div 4 = 42 \text{ r } 1$
The remainder becomes the numerator

Find $\frac{19}{20}$ of 250

Find $\frac{7}{10}$ of 175

Find $\frac{29}{50}$ of 430

B Convert fractions into decimals and percentages

$$\frac{7}{20} = 35\% = 0.35$$

$100 \div 20 = 5$

To find the percentage, multiply the "5" by the numerator

To find the decimal, divide the percentage by 100

More complicated...

$$\frac{7}{12} = 0.583\bar{3}$$

$$12 \overline{) 7.0000}$$

$$\frac{13}{20} =$$

$$\frac{17}{50} =$$

$$\frac{4}{5} =$$

$$\frac{19}{20} =$$

$$\frac{6}{10} =$$

$$\frac{2}{5} =$$

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D Convert improper fractions into fractions

$$\frac{187}{20} = 9\frac{7}{20}$$

The remainder becomes the numerator

The denominator stays the same

Divide the numerator by the denominator. The integer part of this is the whole amount.



$$\frac{148}{20} =$$

$$\frac{12}{9} =$$

$$\frac{27}{16} =$$

$$\frac{83}{16} =$$

$$\frac{26}{12} =$$

$$\frac{37}{15} =$$

$$\frac{67}{25} =$$

$$\frac{35}{25} =$$

$$\frac{23}{8} =$$

E Addition and Subtraction of Fractions with Related Denominators

$$\frac{7}{12} + \frac{5}{6} + \frac{7}{24} = \frac{14}{24} + \frac{20}{24} + \frac{7}{24} = \frac{41}{24} = 1\frac{17}{24}$$

These fractions are related because they are all factors of 24

$24 \div 12 = 2$ so we need to multiply top and bottom by 2.

$24 \div 6 = 4$ so we need to multiply top and bottom by 4.

Once all the denominators are the same, we can simply add the numerators and keep the denominators the same.

Divide the numerator by the denominator. The quotient becomes the whole number and the remainder, the numerator

$$\frac{11}{18} + \frac{5}{9} + \frac{2}{3} =$$

$$\frac{7}{16} + \frac{27}{64} + \frac{13}{32} =$$

$$\frac{7}{8} + \frac{3}{4} + \frac{19}{24} =$$

G Multiplication of Fractions

$$\frac{15}{24} \times \frac{6}{25} = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$$

See if you can cancel the numbers first. Something from the numerators cancels with something from the denominators. You CAN'T cancel denominator with denominator or numerator with numerator.

Multiply top times top. Bottom times bottom.

When you multiply by a fraction, your answer is often smaller than what you began with

$$\frac{8}{20} \times \frac{6}{12} =$$

$$\frac{18}{15} \times \frac{7}{36} =$$

$$\frac{25}{44} \times \frac{22}{24} \times \frac{36}{39} \times \frac{6}{75} =$$

$$\frac{18}{55} \times \frac{45}{64} \times \frac{36}{81} \times \frac{48}{52} =$$

F Addition and Subtraction of Fractions with Unrelated Denominators

$$\frac{11}{12} + \frac{5}{7} = \frac{(11 \times 7) + (12 \times 5)}{(12 \times 7)} = \frac{77 + 60}{84} = \frac{137}{84} = 1\frac{53}{84} \quad \times$$

$$\frac{7}{12} + \frac{5}{11} =$$

$$\frac{5}{9} + \frac{3}{4} =$$

$$\frac{2}{7} + \frac{3}{9} =$$

$$\frac{11}{12} + \frac{3}{8} =$$

$$\frac{6}{11} + \frac{4}{13} =$$

Remember this shape: it tells you what to multiply

H Division of Fractions

$$\frac{7}{20} \div \frac{5}{12} = \frac{7}{20} \times \frac{12}{5} = \frac{7}{5} \times \frac{3}{5} = \frac{21}{25}$$

Keep the first fraction
Change the \div to a \times
Flip the last fraction

Once you have got to the multiplication, you can cancel down.

Top times top
Bottom times bottom

When you divide by a fraction, your answer is often larger than what you began with

$$\frac{18}{21} \div \frac{27}{35} =$$

$$\frac{15}{48} \div \frac{25}{36} =$$

$$\frac{6}{11} \div \frac{7}{12} =$$

$$\frac{16}{27} \div \frac{17}{36} =$$

$$\frac{15}{48} \div \frac{15}{24} =$$

A Ratio

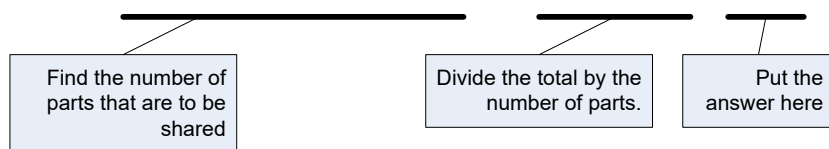
Calculate the number of parts in each of the following ratios.

4:5:6 _____ parts 7:3:8 _____ parts

8:3:12 _____ parts 9:3:8 _____ parts

B Find the value of one part when given a total.

Bill, Julie and Ted share £75 in the ratio 3:4:8. What is the value of one part?



C Find the value of one part when given a total.

Joanne, Neville and Imran share 35 sweets in the ratio 2:1:4. What is the value of one part?

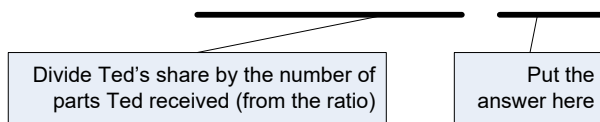
James, Simba and David share £250 in the ratio 9:12:4. What is the value of one part?

Marcus, Jeremy and Paula share 15 shopping bags in the ratio 2:1:2. How many shopping bags are the value of one part?

Charlie, Freda and Samantha share driving out in the ratio 4:5:7. Altogether, they drove 320 miles. What is the value of one share of the driving?

D Find the value of one part when given a single share.

Bill, Julie and Ted share some money in the ratio 3:4:8. Ted got £240. What is the value of one part?



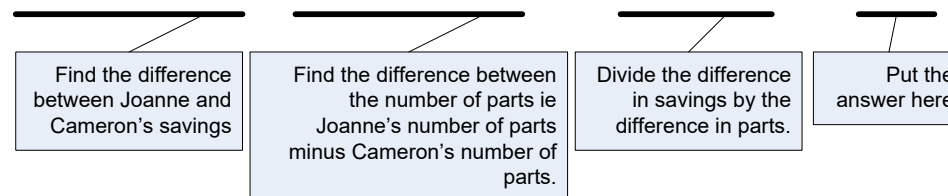
E Find the value of one part when given a single share.

Simon, David and Joshua shared some sweets in the ratio 6:7:4. David got 28 sweets. What is the value of one part?

Matthew, Mike and Sam went for a road trip. They divided the number of hours driving in the proportion 12:14:15. Sam drove for 56 hours. How much was the value of one part?

F Find the value of one part when given a difference between shares.

Nadia, Cameron and Joanne saved some money so that the ratio of their savings was 3:4:7. Joanne saved £12 more than Cameron. How much is the value of one part?

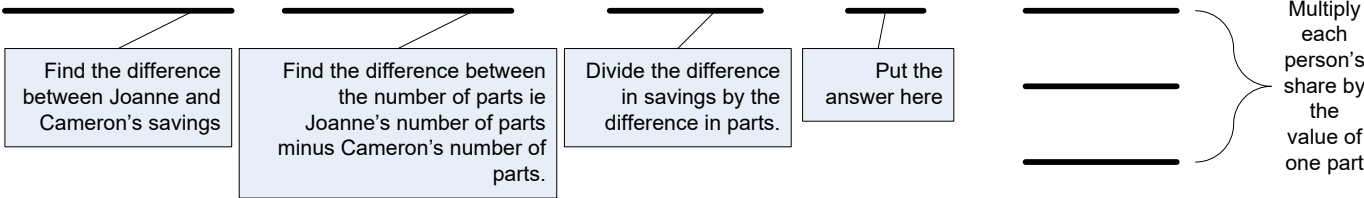


Vicky, Ben and Joshua have a large record collection that they have purchased over the years in the ratio 7:12:13. Ben has 30 more records than Vicky. How many records form one part?



G Find out how much each person has.

Nadia, Cameron and Joanne saved some money so that the ratio of the their savings was 3:4:7. Joanne saved £12 more than Cameron. How much did each person save?



Joanne, Neville and Imran share 35 sweets in the ratio 2:1:4. How many sweets did each person receive?

Matthew, Mike and Sam went for a road trip. They divided the number of hours driving in the proportion 12:14:15. Sam drove for 56 hours. For how long did each person drive?

James, Simba and David share £250 in the ratio 9:12:4. How much did each person share?

Bill, Julie and Ted share some money in the ratio 3:4:8. Ted got £240. How much was each person given?

Simon, David and Joshua shared some sweets in the ratio 6:7:4. David got 28 sweets. How many sweets did Simon and Joshua receive?

H Write these ratios as fractions.

4:5:6

Add up the parts. This becomes the denominator.

6:9:22

11:26:19

12:23:41

Marcus, Jeremy and Paula share 15 shopping bags in the ratio 2:1:2. How many shopping bags did each person carry to the car?

Vicky, Ben and Joshua have a large record collection that they have purchased over the years in the ratio 7:12:13. Ben has 30 more records than Vicky. How many records had each person purchased?

Nadia, Cameron and Joanne saved some money so that the ratio of the their savings was 3:4:7. Joanne saved £12 more than Cameron. How much had each person saved?

Charlie, Freda and Samantha share driving out in the ratio 4:5:7. Altogether, they drove 320 miles. For how many miles did each person drive?

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Exchange Rates £1 : 1.70 Sin\$

I swap £450 into Singapore dollars. How much do I get?

I have Sin\$185 to change back into GB£. What do I get?

Multiply the amount by the exchange rate that you are changing the money into.

Divide the amount by the exchange rate of the money you are changing from.

I go to Australia with £1250. The exchange rate is £1:1.85 AU\$. I spend AU\$ 2150. How much money in GB£ do I return home with?

I go to the USA with \$4000. The exchange rate is £1:1.19 US\$. How much GB£ did I need to purchase this amount? I spent \$3650. How many GB£ did I have remaining?

Simplify the following ratios

1. 8:4
2. 12:9
3. 15 : 20 : 35
4. 18 : 24 : 48
5. 21: 14: 28

Write the following in the form 1: n or 1:n:m

1. 8 : 2
2. 12 : 40
3. 5 : 7
4. 8 : 24 : 12
5. 10 : 14 : 28

Write the following in the form A:B:C

1. A:B s 3:2. B:C is 5:4
2. A:B is 8:5 B:C is 7:4
3. A:B is 9:2 A:C is 6:5
4. A:B is 9:4 B:C is 8:7
5. A:B is 11:5 A:C is 9:4

Proportion

1. 8 apples cost 40p. How much for 5 apples?
2. A recipe uses 350g flour for 5 buns. How much is needed for 11 buns?
3. 6 pens cost £4.80. How much do 11 pens cost?
4. 300g of sweets cost £1.44. How much for 1kg?
5. 4 men take 7 hours to do some work. How long would it take 3 men?
6. It takes three hosepipes 4 hours to fill a pond. How long would it take five hosepipes?

Exchange Rates

1. £1:\$1.25 How many \$ for £250?
2. £1: R 24 How many £ for R500?
3. £1: AU\$2.15 How many £ for AU\$600?
4. \$1: £0.71 How many £ for \$650?
5. £1 : \$1.43 How many £ for \$450?
6. £1 : AU\$2.43 How many £ for AU\$450?

Best Value

1. Bill wants to travel 16 miles.
Taxi A £3.00 + £1.20 per mile
Taxi B £6.50 + £0.98 per mile
Taxi C £10.00 + £0.76 per mile
Which taxi should he use and how much would it cost?
2. Mary wants to go shopping. She hasn't got a car so chooses to go by bus to one place. She wants to buy two legs of lamb and twelve tins of soup.
Centre A Lamb: £11.20 per leg
Soup: £1.45 per tin
Centre B Lamb: £14.00 per leg
Soup: £0.98 per tin
To which shopping centre should she go?
3. The Smiths want to go on holiday. There are four of them. They choose to go for seven nights.

Travel Agent A
Return Flight £284.95 each.
Hotel £78.95 per person per night

Travel Agent B
Return Flight £437.55 each.
Hotel £57.14 per person per night

With which travel agent should the Smith family book their holidays?

How much did each person get?

1. Bill and John share an inheritance in the ratio 4:7. The inheritance was £781. How much did each person receive?
2. Naveed, Akash and Nimrah receive some money for Eid. The money they receive is in the proportion 7:3:6. Naveed gets £161. How much do the other two get?
3. Ian, Joanne and Mary buy some shares in the proportion 8:11:4. Joanne pays £954 more than Ian on the shares. How much does each person pay for the shares?
4. Neil, Simon and Matthew went out for a curry. Altogether, the bill came to £128. They put in all the money they had with them in the ratio 4:5:7. How much did each person pay?
5. Martin, Vicky and Mark all go on holiday with their families. The size of their families are in the ratio 5:3:4. The amount they spent on their holidays per person was in the ratio 4:7:6. If Vicky spent a total of £396.90, how much did Martin and Mark spend?

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Probability

1. The ratio of throwing heads or tails on a biased coin is 6:5. Bill threw the coin 550 times. How many times would you expect it to come up heads?
2. The ratio of winning to losing at a game of cards are 3:7. What are the chances of winning four games in a row?
3. A dice has a 0.3 chance of landing on 6. The chances of rolling any number from 1 to 5 are the same as each other. What is the ratio for rolling a 6 to rolling an odd number?
4. Bill and Ted were investigating prime and composite numbers. They split all the numbers from 1 to 200 into blocks of 20 (ie 1 – 20, 21 – 40, 41 – 60 and so on).

Bill said, "As the number of the block gets higher, the ratio of prime to composite gets smaller."

For each block, work out the ratio in the form 1:n to determine whether Bill was correct.

Ratio and Proportion

Write these ratios as fractions

1. 8 : 4 : 9
2. 15 : 7 : 11
3. 15 : 20 : 35 : 18
4. 18 : 24 : 48 : 8 : 12
5. 42 : 14 : 28 : 15 : 16

Write these fractions as ratios in their simplest form

1. $\frac{16}{25}$ $\frac{45}{50}$
2. $\frac{3}{10}$ $\frac{4}{25}$
3. $\frac{28}{35}$ $\frac{15}{50}$
4. $\frac{62}{100}$ $\frac{84}{200}$

Midpoints and Endpoints

A line segment AB runs from (3,7) to (5,15). What are the co-ordinates of the following points.

- a The midpoint
- b The point at a ratio of 2:3 along AB
- c The point at a ratio of 4:8 along AB

G Find possible numbers given the HCF and LCM

There are various answers for these types of question.

A and B are two integers.

The $HCF(A,B) = 18$ and the $LCM(A,B) = 648$.

What could A and B be?

1. Draw a factor tree for the HCF and LCM

2. Cross out the prime factors matching the "18" of the HCF.

3. Put the factors from the HCF in the intercept section of the Venn diagram.

4. The remaining numbers from the LCM can be placed where you like, outside the intersect, in the Venn diagram.

5. Multiply the numbers in circle A and then multiply the numbers in circle B.

$A = 2 \times 2 \times 2 \times 3 \times 3 = 72$
 $B = 2 \times 3 \times 3 \times 3 \times 3 = 162$

$HCF(C,D) = 12$
 $LCM(C,D) = 40$
 $HCF(E,F) = 15$
 $LCM(E,F) = 135$

H Apply the HCF and LCM.

Two LEDs are flashing. LED A flashes every 8 seconds. LED B flashes every 34 seconds. At precisely 12 noon, both LEDs flash together. At what time do they next flash together?

Two friends work on the buses. Fred drives route A which takes 54 minutes. Leanne drives route B which takes 36 minutes. They both set off at precisely 9am on their respective routes. At what time do they next see each other at the bus station?

Billy wants to tile a wall that is 240 cm high and 345 cm long. He hates cutting tiles as they always seem to snap so wants to tile his wall with whole tiles. What is the largest size of tile he can use and how many tiles will he need?

I Using HCF and LCM in algebra

Factorise $8p^2q^3r^5 + 36p^3qr^2$

$$8p^2q^3r^5 = 2 \times \cancel{2} \times \cancel{2} \times p \times p \times q \times q \times q \times \cancel{r} \times \cancel{r} \times r \times r \times r$$

$$36p^3qr^2 = \cancel{2} \times \cancel{2} \times 3 \times 3 \times p \times p \times p \times q \times \cancel{r} \times \cancel{r}$$

1. Split each expression into its prime factors.

2. Cross out the common prime factors and write them into the intersect section of the Venn diagram.

3. Put the remaining factors into the outside part of the Venn diagram making sure you place them in the correct circle.

4. Reconstitute the expressions from the prime factors

5. Put the intersect outside the brackets.

6. Make sure you use the same operator ie + or - as was used in the original question.

$2 \times q \times q \times r \times r \times r = 2q^2r^3$
 $3 \times 3 \times p = 9p$
 $2 \times 2 \times p \times p \times q \times r \times r = 4p^2qr^2$
 $4p^2qr^2(2q^2r^3 + 9p)$

Factorise $16x^2y^3z^5 + 40x^3yz^2$ Factorise $18t^5v^3w + 66t^2w - 12$

Factorise $35c^5d^2e + 75c^3d^2$ Factorise $48k^2r + 64k^5t - 6t^2$

Simultaneous Equations

A Choose the variable you want to keep.

$$\begin{aligned} 3x + 8y &= 74 \\ 3x + y &= 25 \end{aligned}$$

The x coefficient (number in front of the x) is the **same** in both cases. You can take one from the other leaving just the y part of the equation. It doesn't matter if the number is negative or positive at this stage.

$$\begin{aligned} 4 \quad 9x + 3y &= 9 \\ 8x - 3y &= 25 \end{aligned}$$

$$\begin{aligned} 5 \quad 4x + 3y &= 32 \\ 8y - 4x &= 24 \end{aligned}$$

$$\begin{aligned} 1 \quad 6x + 5y &= 74 \\ 6x + 2y &= 25 \end{aligned}$$

$$\begin{aligned} 2 \quad x + 4y &= 20 \\ 3x + 4y &= 36 \end{aligned}$$

$$\begin{aligned} 3 \quad 5x + y &= 37 \\ 3x + y &= 23 \end{aligned}$$

$$\begin{aligned} 6 \quad 9x + 5y &= 53 \\ 4y - 9x &= 10 \end{aligned}$$

$$\begin{aligned} 7 \quad 3(7x + 3y) &= 189 \\ 9y - 12x &= -9 \end{aligned}$$

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B Add or subtract the equations

$$\begin{aligned} 3x + 8y &= 74 \\ 3x + y &= 25 \end{aligned}$$

Here, the "signs" for the coefficients in front of the variable you want to remove are the **same** (ie both positive) so you need to **subtract** the equations.

$$\begin{aligned} 9x + 5y &= 53 \\ 4y - 9x &= 10 \end{aligned}$$

Here, the "signs" for the coefficients in front of the variable you want to remove are the **different** (ie positive and negative) so you need to **add** the equations so the x terms cancel each other out.

$$\begin{aligned} 1 \quad 6x + 5y &= 74 \\ 6x + 2y &= 25 \end{aligned}$$

$$\begin{aligned} 3 \quad 9x + 3y &= 9 \\ 8x - 3y &= 25 \end{aligned}$$

$$\begin{aligned} 5 \quad 4x + 3y &= 32 \\ 8y - 4x &= 24 \end{aligned}$$

$$\begin{aligned} 2 \quad x - 4y &= 2 \\ 3x - 4y &= 22 \end{aligned}$$

$$\begin{aligned} 4 \quad 9x + 5y &= 53 \\ 4y - 9x &= 10 \end{aligned}$$

$$\begin{aligned} 6 \quad 3(7x + 3y) &= 189 \\ 9y - 12x &= -9 \end{aligned}$$

C Find the value of the first variable

$$\begin{aligned} 3x + 8y &= 74 \dots\dots (i) \\ 3x + y &= 25 \dots\dots (ii) \end{aligned}$$

Name the equations so we can refer to them easily

Subtract (i) – (ii)

$$\begin{aligned} (3-3)x + (8-1)y &= 74 - 25 \\ 7y &= 49 \\ y &= 7 \end{aligned}$$

We subtract because the x-terms (that we are trying to get rid of) are both the same sign (ie positive). Sometimes we do (ii) – (i)

$$\begin{aligned} 1 \quad 6x + 5y &= 74 \\ 6x + 2y &= 25 \end{aligned}$$

$$\begin{aligned} 2 \quad x + 4y &= 20 \\ 3x + 4y &= 36 \end{aligned}$$

$$\begin{aligned} 3 \quad 5x + y &= 37 \\ 3x + y &= 23 \end{aligned}$$

Find the first variable with these equations



D Find the value of the second variable

$$\begin{aligned} 3x + 8y &= 74 \dots\dots (i) \\ \vdots & \end{aligned}$$

We have worked out that y=7 in part C.

Substitute in (i)

$$\begin{aligned} 3x + 8(7) &= 74 \\ 3x &= 74 - 56 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

We take the value for y (which we found in part C) and substitute it into this equation.

You can check your answer by substituting into equation (ii) to see if it works. If it does... well done

Now answer questions 1 – 7 given in **A**

Simultaneous Equations

E What if the coefficients are related but not the same?

$$3x + 8y = 106 \dots\dots (i)$$

$$x + 2y = 28 \dots\dots (ii)$$

Neither the x nor the y coefficients are the same in this pair of equations. You may notice that if you multiply equation (ii) $\times 4$, you will get a y coefficient of 8 in both equations. You can then continue as before.

Decide which equation you are going to multiply, and by how much

1	$6x + 5y = 119$
2	$x + 9y = 179$
3	$9x + y = 117$

$$2x + 2y = 44$$

$$3x + 4y = 123$$

$$3x - 4y = -39$$

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$$3x + 8y = 106 \dots\dots (i)$$

$$x + 2y = 28 \dots\dots (ii)$$

Multiply (ii) $\times 4$

$$4x + 8y = 112 \dots\dots (iii)$$

Subtract (iii) - (i)

$$(4-1)x + (8-8)y = 112 - 106$$

F What if the coefficients are unrelated and not the same?

$$7x + 8y = 43 \dots\dots (i)$$

$$5x + 3y = 47 \dots\dots (ii)$$

Here, the coefficients are unrelated: 8 is not a multiple of 3; 7 is not a multiple of 5. You decide which term, x or y, you wish to get rid of. Imagine we have chosen, "Get rid of x". Multiply equation (i) $\times 5$ (which is the x coefficient in equation (ii)). Then, multiply equation (ii) $\times 7$ (which is the x coefficient in equation (i)). If you had decided on y, you would multiply (i) $\times 3$ and (ii) $\times 8$.

Multiply (i) $\times 5$, (ii) $\times 7$

$$35x + 40y = 215 \dots\dots (iii)$$

$$35x + 21y = 329 \dots\dots (iv)$$

Subtract (iii) - (iv)

⋮

1	$7x + 8y = 43$
2	$5x + 9y = -9$
3	$8x + 7y = 67$

$$5x + 3y = 47$$

$$3x + 4y = 3$$

$$3x - 4y = 98$$

Find equations (iii) and (iv)

G Solve the equations in their entirety.

$$9x + 8y = 47 \dots\dots (i)$$

$$5x - 3y = 41 \dots\dots (ii)$$

Multiply (i) $\times 5$, (ii) $\times 9$

$$45x + 40y = 235 \dots\dots (iii)$$

$$45x - 27y = 369 \dots\dots (iv)$$

Subtract (iii) - (iv)

$$(45-45)x + (40-(-27))y = 235-369$$

$$67y = -134$$

$$y = \frac{-134}{67}$$

$$y = -2$$

Be careful with double minuses etc.

Substitute in (i)

$$9x + 8(-2) = 47$$

$$9x - 16 = 47$$

$$9x = 47 + 16$$

$$9x = 63$$

$$x = 7$$

Check in (ii)

$$5x - 3y = 41 \dots\dots (ii)$$

$$5(7) - 3(-2) = 41$$



1	$11x + 6y = 30 \dots\dots (i)$
2	$4x - 7y = -355 \dots\dots (i)$
3	$8x + 5y = 81 \dots\dots (i)$
4	$7x + 4y = 43 \dots\dots (i)$
5	$2x + 7y = 5.2 \dots\dots (i)$
6	$7x + 6y = 229 \dots\dots (i)$

$$5x - 5y = 60 \dots\dots (ii)$$

$$8x - 5y = -161 \dots\dots (ii)$$

$$2x - 5y = 89 \dots\dots (ii)$$

$$9x + 3y = 21 \dots\dots (ii)$$

$$12x - 4y = 12.8 \dots\dots (ii)$$

$$9x - 4y = -303 \dots\dots (ii)$$

Equal coefficients

This may involve finding one unknown in each equation with the same coefficients or multiplying one of more of the equations to manufacture the same coefficients.

Get rid of one unknown

This involves either adding (if the coefficients are of different signs (- or +) or subtracting if they are the same sign.

Substitute into equation (i) or (ii)

Swap the value you have found for x or the value you have found for y into equation (i) or (ii). Solve the equation.

Check your answer

Swap the value you have found for x or the value you have found for y into the other equation. If it works - give yourself a tick!!

Note the strategy for solving simultaneous linear equations

Answers to part G

1 $x=6$ $y=-6$

2 $x=18$ $y=61$

3 $x=17$ $y=-11$

4 $x=-3$ $y=16$

5 $x=1.2$ $y=0.4$

6 $x=-11$ $y=51$